# Context-Enhanced Vessel Prediction Based On Ornstein-Uhlenbeck Processes Using Historical AIS Traffic Patterns: Real-World Experimental Results

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***Abstract*—Traffic route analysis and prediction are both essential for maritime security. Specifically, the prediction of a vessel position is useful to provide alerts about upcoming events (e.g., opportunities and threats). However, accurate prediction along a route in the maritime domain is a challenging task, due to the complex and dynamic nature of traffic patterns.**

**This paper presents a vessel prediction method, based on the popular Ornstein-Uhlenbeck stochastic processes, whose parameters are estimated from historical patterns of life. The historical traffic routes are obtained by pre-processing Automatic Identification System (AIS) data via the CMRE tool called Traffic Route Extraction for Anomaly Detection (TREAD).**

**These recurrent routes allow prediction of the position of a vessel that is following one of these routes, surprisingly, by several hours. The method is validated using a case study related to the second data campaign of the EC FP7 Project *New Service Capabilities for Integrated and Advanced Maritime Surveillance* (NEREIDS)[1]. We demonstrate that the prediction accuracy is well represented by the Ornstein-Uhlenbeck model.**

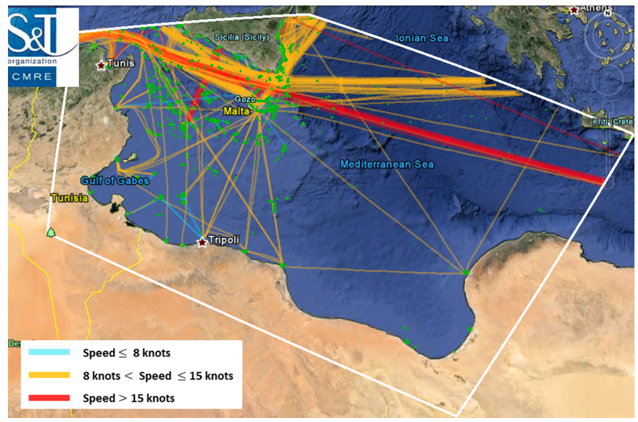


Fig. 1. TREAD historical route atlas from AIS data over the South Mediterranean sea. Routes are directional and colour-coded based on the observed speed profile along each route.

## I. Introduction

Maritime transportation is by volume the most utilized form of transportation supporting the global supply chain, making maritime safety and security an important concern. A key aspect of maritime safety and security is Maritime Situational Awareness (MSA), which is achieved through surveillance and tracking. Cooperative vessel self-reporting systems, including the AIS, provide a vast amount of near real time information [1], [2], requiring an ever increasing degree of automation in order to transform data into decision support elements.

AIS transponders transmit on VHF frequencies of 161.975 Mhz and 162.025 Mhz. Furthermore, there are two classes of transmitters: A (minimum 12 Watts) and B (minimum 2 Watts). In this analysis, there is no distinction made between detections from A or B class transmitters. Each AIS transmitting vessel will report its position depending on factors such as the speed and manoeuvring status. Class A transmitters transmit at least every 3 minutes and up to every 2 seconds. Class B transmitters typically transmit every 30 seconds.

While the AIS system was originally conceived for collision avoidance, and the main use of the system is for local and real time applications, there are increasing possibilities for the use of AIS beyond this scope. Coastal states are also able to receive, plot and log the data by means of receiving stations along the coast, in the air, or in space. The AIS information can also be used as ground truth in order to estimate the performance of coastal radars, see e.g. [3]–[7].

Previous work for the automated learning of vessel traffic routes [8], [9] has shown that through the analysis of this historical data, valuable knowledge about the behaviour of maritime traffic can be extracted. This previous work has culminated in the development of the CMRE tool called Traffic Route Extraction for Anomaly Detection (TREAD). A representation of the maritime traffic routes obtained using TREAD is shown in Fig. 1.

Once the picture of the maritime traffic is derived, this historical knowledge can be applied to:

- classify the routes, *i.e.*, identify compatible routes, assigning a probability that the vessel is actually following a specific route;
- predict future vessel positions, *i.e.*, predict the route along which a vessel is going to move, in agreement with the partially observed track and given the vessel static information (*e.g.*, vessel type);
- support decisions and action planning at different information levels, enhancing the detection of anomalous behaviours, *i.e.*, behaviours deviating from the learned traffic normality;

[1] http://www.nereids-fp7.eu/ - NEREIDS project website

- optimize sensor resource management, *i.e.*, the problem of allocating the available sensor resources in order to obtain the optimal awareness of the situation.

Some of these potential applications are being investigated within the EC FP7 Project NEREIDS consortium, where CMRE contributed to the multi-sensor data fusion module and the route propagation module [10].

This paper presents a vessel prediction method, based on the Ornstein-Uhlenbeck [11], [12] stochastic processes, whose parameters are estimated from historical AIS data. These parameters are essential characteristics of recurrent routes, and can be exploited as prior knowledge in order to predict the position of vessels with a given confidence on the error estimates. An alternate approach to vessel motion prediction from AIS data is via a particle filtering approach presented in [13]. In the particle filtering case, the main aim of the study was to detect anomalies in vessel motion assuming that the recurrent patterns of life are already derived and correctly represent the traffic normality.

In the method proposed here, it is demonstrated that the prediction errors are well represented by the Ornstein-Uhlenbeck model using real-world AIS data categorized into routes. While the particle filtering approach can provide complex (*i.e.* multi-modal) probability densities for prediction, the advantage of the stochastic model presented here is the relative simplicity of the modelling and parametrization of individual traffic routes. Possible multi-modalities in the stochastic approach can be later derived by superimposing the predictions along multiple compatible routes.

The remainder of the paper is organized as follows. Sections II and III review related work in the field of traffic analysis and give an overview of the proposed approach. Section IV applies the methodology to a real data set, and conclusions are reported in Section V.

## II. Traffic Route Extraction

Comprehensive knowledge of recurrent vessel patterns in an area under investigation is valuable information to support accurate vessel predictions. However, the increasing quantity of historic AIS reports poses new challenges in the related fields of data mining and machine learning techniques when applied within the context of big data and MSA. The large amount of vessel movement data collected by terrestrial networks and satellite constellations of AIS receivers requires the aid of automatic processing techniques to fully exploit this data, since the initial amount of raw information can overwhelm human operators. Also, the learning process should be robust with respect to number of sensors, their coverage and refresh rate, and scale of the area of interest. Thus, it is desirable to base the traffic route extraction process on incremental learning which can be applied both in real-time or batch fashion.

Within this context, traffic route extraction and prediction in an unsupervised way, based on historical knowledge of maritime traffic, is an essential first step. It implies that raw maritime data can be translated into information. Although AIS data reliably depicts the traffic related primarily to large vessels, it can be effectively used to infer different levels of contextual information, spanning from the characterization of ports and off-shore platforms to spatial and temporal distributions of routes. The proposed methodology exploits the knowledge about the traffic patterns in the area of interest derived using the CMRE TREAD tool. This tool automatically learns a synthetic representation of maritime traffic patterns from low-level AIS data in an unsupervised way. More details about the TREAD tool can be found in [8], [9], [14].

The knowledge achieved via TREAD about the traffic scene is shaped in a compact form via three mutually dependent classes of objects: *vessels*, *waypoints* and *routes*. These objects are generated by incremental processing of raw data where meaningful events are generated in the vessel state vector sequences, including events like a break in observation updates. Specifically, the recurrent patterns of life are derived by clustering these meaningful events in the vessel behaviour, enabling the creation of waypoints of interest which can be classified as either *stationary areas* ($POs$), *entry points* ($ENs$) and *exit points* ($EXs$) within the selected bounding box. *Routes* ($Rs$) are then created by connecting such *waypoints* and are statistically characterized.

This automated route-extraction technique is useful by itself to inform the understanding of patterns of life in the maritime domain and to enhance MSA applications (for defence and commercial purposes). One potential application is the sensor resource management, where predictive models are needed, able to approximate the expected movement of the targets in general (and, more specifically, of vessels). To this aim, the derived traffic information can be exploited by assuming that anomalies are very unlikely to be observed for the vessels of interest, and the route prediction can be performed by context-based tracking algorithms.

Information from the derived routes, such as the vessel type, mean velocity or the series of route points provided by previous transits, represent a set of constraints that can be used to characterize the behaviour of specific classes of vessels along the route. Together, the traffic routes and route characteristics can be used in the prediction of vessel positions with the aid of non-linear filtering techniques. The following section will illustrate how these key concepts can be fed into the proposed context-based tracking model to effectively predict vessel position in the future.

## III. Stochastic Model of the Vessel Kinematics

In this section we describe the stochastic model of the vessel kinematics, given that a vessel belongs to a specific route. Moreover, the generic term *target* will be adopted herein instead of *vessel*, in order to conform with the related tracking literature. The target-to-route association process is out of the scope of this work, however useful entry points about this problem are given in [15]–[19]. In the context of ground target tracking, of particular interest is the Variable Structure Interactive Multiple Model (VS-IMM) mechanism [15], [16], that is used to handle the on/off-road transitions and the change from one road to another. Transition probabilities for the routes are used in [19], where the probability has been shown to be well described by a Weibull distribution.

The objective is to describe, given the hypothesis that a target is following a route, how the state of the target and its related uncertainty will evolve over time.

The target state at time $t \in \mathbb{R}$ is indicated with $\boldsymbol{x}(t) = [x(t), y(t), \dot{x}(t), \dot{y}(t)]^T$, where the two coordinates are $\boldsymbol{x}_p(t) \stackrel{def}{=} [x(t), y(t)]^T$, and $\dot{\boldsymbol{x}}_p(t) \stackrel{def}{=} [\dot{x}(t), \dot{y}(t)]^T$ are the corresponding velocities. It is assumed that the target dynamics are given by a set of linear stochastic differential equations (SDE) of the form:

$$d\boldsymbol{x}(t) = \mathbf{A}\;(\boldsymbol{x}(t) - \boldsymbol{m})\;dt + \mathbf{B}\,d\boldsymbol{w}(t), \qquad (1)$$

where $\boldsymbol{m} = \left[0, 0, \boldsymbol{v}^T\right]$, and $\boldsymbol{w}(t)$ is a standard bi-dimensional Wiener process. The matrices $\mathbf{A}$ and $\mathbf{B}$ are defined as:

$$\mathbf{A} = \left[\begin{array}{cc} \mathbf{0} & \boldsymbol{I} \\ \mathbf{0} & -\gamma \boldsymbol{I} \end{array}\right], \mathbf{B} = \left[\begin{array}{cc} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{C} \end{array}\right], \qquad (2)$$

where $\mathbf{0}$ is bi-dimensional matrix of null elements, $\boldsymbol{I}$ is the identity matrix, $\gamma = [\gamma_x, \gamma_y]^T$ is a vector whose elements are positive values, and $\mathbf{C}$ is a bi-dimensional matrix.

In general, SDEs can be solved in closed form by using Itô calculus ( [20]). In our case, equation (1) have the form of a Langevin dynamic [21], and the $\dot{\boldsymbol{x}}_p(t)$ process is said to be of Ornstein-Uhlenbeck type [11], [12]. Correspondingly, we say that $\boldsymbol{x}(t)$ is an integrated Ornstein-Uhlenbeck process [12].

Ornstein-Uhlenbeck processes are very popular in several areas of research: finance ( [12], [22]), physics ( [11], [20], [21]) and network control [23]. These processes are sometimes used also in the target tracking community, see e.g. [24]–[30], to model the autocorrelation of acceleration. However note that, due to the nature of the radar or sonar tracking problem, the continuous-time models, similar to (1), are discretized in time steps $t_k$. However, given the asynchronous nature of the AIS data, this discretization is avoided in this work and the target state is modelled as continuous-time process.

The parameters $v_x$ and $v_y$, in $\boldsymbol{v} = [v_x, v_y]^T$, play a key role in the proposed model because they represent the *typical* velocities, along $x$ and $y$ respectively, of the route under consideration. These velocities can be empirically estimated by using historical routes which provide a reference mean transit speed, as shown in the experimental section. Assuming that the target is associated to the route at the time $t_0$, then we have that its expected position is propagated at constant velocity $\boldsymbol{v}$, so we can write as in [12], [20]:

$$\mathbb{E}\left[\boldsymbol{x}(t)\middle|\boldsymbol{x}(t_0)\right] \approx [\boldsymbol{x}_p(t_0) + t\,\boldsymbol{v}, \boldsymbol{v}]^T\,. \qquad (3)$$

This approximation derives from the fact that we neglect the initial transitory phase (vanishing exponentially in $t$). Define $t^* = t - t_0$ as the time window, and the related variances are then given by:

$$\text{VAR}\left[x(t)\middle|\boldsymbol{x}(t_0)\right] = \frac{\sigma_x^2}{2\gamma_x^3}\left(2\gamma_x t^* + 4e^{-\gamma_x t^*} - e^{-2\gamma_x t^*} - 3\right) \quad (4)$$

$$\text{VAR}\left[y(t)\middle|\boldsymbol{x}(t_0)\right] = \frac{\sigma_y^2}{2\gamma_y^3}\left(2\gamma_y t^* + 4e^{-\gamma_y t^*} - e^{-2\gamma_y t^*} - 3\right) \quad (5)$$

$$\text{VAR}\left[\dot{x}(t)\middle|\boldsymbol{x}(t_0)\right] = \frac{\sigma_x^2}{2\gamma_x}\left(1 - e^{-2\gamma_x t^*}\right) \qquad (6)$$

$$\text{VAR}\left[\dot{y}(t)\middle|\boldsymbol{x}(t_0)\right] = \frac{\sigma_y^2}{2\gamma_y}\left(1 - e^{-2\gamma_y t^*}\right) \qquad (7)$$

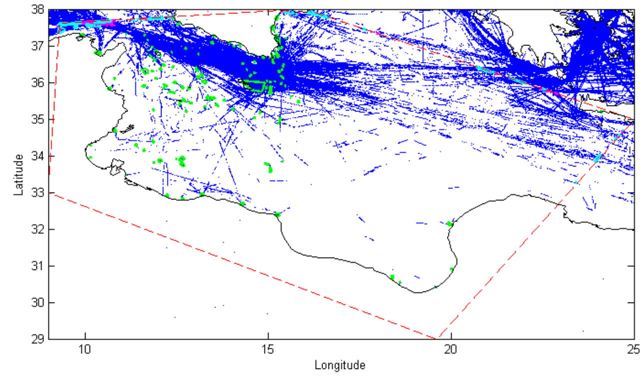


Fig. 2. AIS data (blue dots) for the last ten day window in the area of interest collected by the MSSIS sensor network.Time-frame: May 1 to Sept 10, 2012. TREAD unsupervised analysis led to the detection of Entry (cyan), Exit (magenta) and Stationary areas (green).

where $\sigma_x^2$ and $\sigma_y^2$ are the entry elements of $\text{diag}\left(\mathbf{C}\mathbf{C}^T\right)$.

The parameters $\boldsymbol{v}$, $\gamma$ and $\sigma_{x,y}$ fully characterize the statistical properties of the route under consideration. In particular, assuming that we observe the target state at a given time $t_0$, we can predict its position and velocity by using (3), whose uncertainty is given by eqs. (4-7). A key element of the proposed methodology, that could be in contrast with the common models adopted in the tracking literature (see for instance [15]), is that the variance of the target position grows linearly with time, $\sim \sigma_{x,y}^2/\gamma_{x,y}^2\,t$ (see eqs. (4-5)). In the popular *near constant velocity* model, see [26] the target position variance grows faster $\sim \tilde{\sigma}^2\,t^n$ with $n \geq 3$. Clearly, in most tracking applications this is not a problem because the data rate is quite large (e.g., in sonar systems, less than a minute [31]) if compared to our case in which we aim at providing an estimated vessel prediction of several hours. However, we found that the correct scaling law of the variance is linear and in the next section we validate the corresponding Ornstein-Uhlenbeck model using a real-world data set.

## IV. Real-world Experimental Results

The NATO Science and Technology Organization Centre for Maritime Research and Experimentation (STO-CMRE) collects historical AIS data from the MSSIS sensor network and uses this data for research purposes[2]. The terrestrial AIS data collected using the MSSIS network are here used to validate the proposed model presented in Section III.

### A. Raw Traffic Data and Pre-processing

The selected bounding box covers an area of about 400 x 600 nautical miles in the South Mediterranean Sea, as shown in Fig. 2. This is the area of the second data campaign within the EC FP7 project NEREIDS. The dataset covers a time period going from 1 May to 10 September, 2012. Fig. 3 summarises the overall distribution of vessel types in the analysed area.

[2]MSSIS is an unclassified government-to-government near real-time data collection and distribution network for AIS data, based on the contributions of a global network of member nations. MSSIS is mainly a shared network of coastal AIS receivers. MSSIS data were not part of the NEREIDS campaign but were used to complement the data fusion, the optimization of data collection during the second data campaign, and the historical traffic analysis.

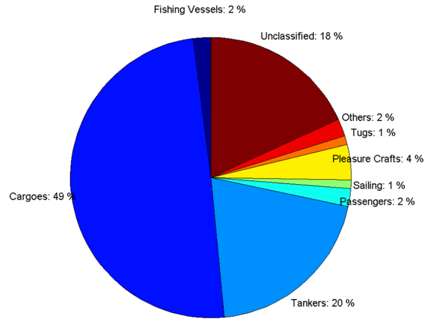


Fig. 3. Vessel type distribution as derived from the AIS data in the area of interest for the given time-frame

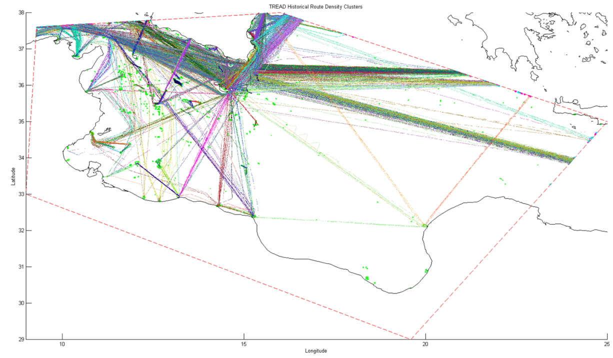

Fig. 4. TREAD output: Extraction of color-coded clusters of historical routes.

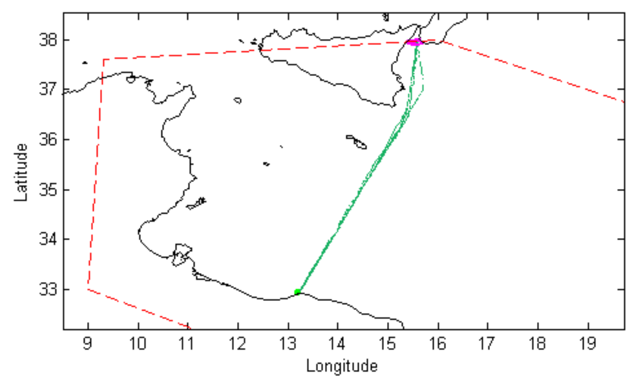


Fig. 5. Cargo route cluster: mean transit speed $= 14.35$ knots, route length $= 601$ km, median transit duration $= 21.7$ h.

These historic AIS data have been used to extract the recurrent patterns of life via TREAD analysis, useful to support the real-time data acquisition during the campaign by providing information about the areas with higher vessel traffic density. Since the MSSIS sensor coverage in the area is not optimal, considerable effort was put into filtering out the portion of inconsistent contacts resulting from incorrect transmission of timestamps and positions (as a consequence of asynchronous receivers).

The dataset obtained from this pre-processing phase was then used to test the proposed model. The data were converted from latitude and longitude to Cartesian $(x, y)$ coordinates in Universal Transverse Mercator (UTM). The adoption of UTM coordinates is fairly frequent in the case of AIS data ( [32]). Note that the error related to the transformation is *relatively* negligible given that predictions of several hours will have substantially larger errors.

### *B. Entry/Exit Points and Stationary Areas*

A first-level contextual information layer derived from the dataset via TREAD analysis is represented by *waypoints*, which include *entry* (cyan), *exit* (magenta) and *stationary areas* (green) (as shown in Fig. 2, along with last ten-day raw AIS data depicted as blue dots). It is noteworthy to recall that *stationary areas* can be either *ports*, off-shore *platforms*, *anchorage areas* or *fishing areas*, depending on the specific statistics of the vessels which contributed to form these clusters.

The traffic *routes* in the area are generated by connecting the derived *waypoints* using associated detections of vessels connecting the *waypoints*.

### *C. Historical Route Codebook*

A second-level contextual information layer derived via TREAD is the set of filtered colour-coded route clusters summarised in Fig. 4.

Furthermore, to ease maritime surveillance over very busy areas, these route clusters can be represented in a compact way, by deriving synthetic routes (as discussed in [8]), which synthesize the *average* behaviour along the route and store statistics about the static and dynamic features of the vessels which transited along them. The resulting atlas of historical routes provides a codebook of traffic activity in the area of interest, which can be categorized by speed, as shown in Fig. 1.

Each route corresponds to a *typical* motion behaviour between two given *waypoints*, and can be used as the input motion information for the proposed Ornstein-Uhlenbeck tracking model.

In order to verify the model, each derived route, which is representative of a motion profile, has been decomposed into piece-wise continuous trajectories of the vessels which transited along it. These trajectories were used to investigate the variability of the bivariate *prediction error* defined as the displacement of the current observed state $\boldsymbol{x}(t)$, with respect to the predicted one $\hat{\boldsymbol{x}}(t)|\boldsymbol{x}(t_0)$, given a time interval, $t^*$. More specifically, the accuracy in prediction has been studied by calculating empirically the variance of the errors in position along the two Cartesian dimensions on a range of time intervals used for the prediction, and considering $t_0$ equal to zero.These estimated variances have then been compared with the models for position presented in eqs. 4-5.

In the following sub-sections, the results obtained by testing the model in three illustrative cases (which differ for the ship-type distribution and, hence, for the speed profile) are presented.

#### *Example 1: Cargo Route*

Figure 5 depicts the selected cargo route. Some additional statistics are also reported.

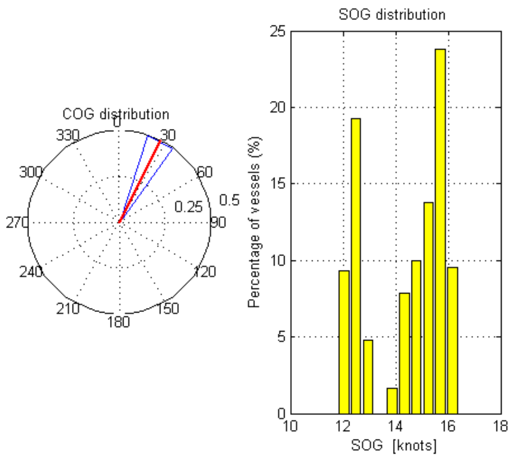


Fig. 6. Cargo route statistics: SOG distribution and COG distribution

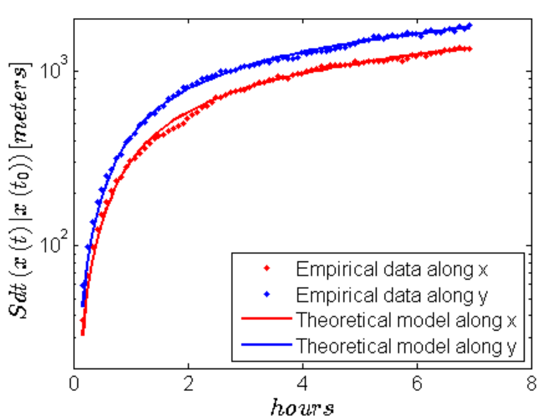


Fig. 7. Visualization of prediction error standard deviation along x and y for the cargo route

*Example 2: Tug Route*

This section reports the results for a shorter route, transited by tugs.

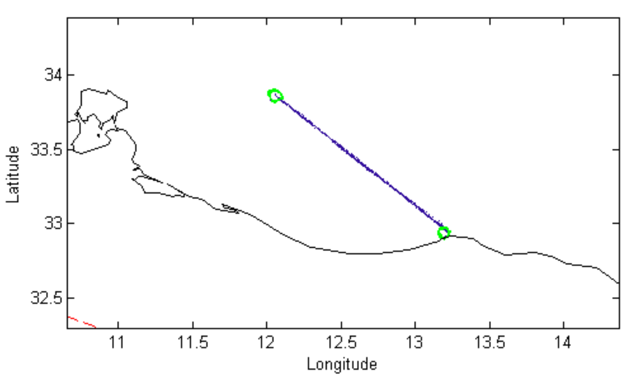


Fig. 8. Tug route cluster: mean transit speed = 8.28 knots, route length = 140 km, median transit duration = 10.6 h.

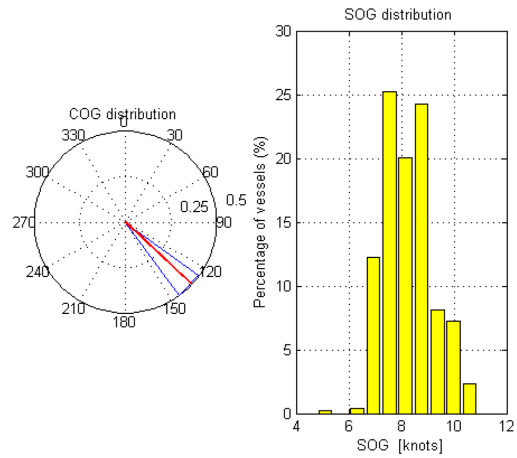


Fig. 9. Tug route statistics: SOG distribution and COG distribution

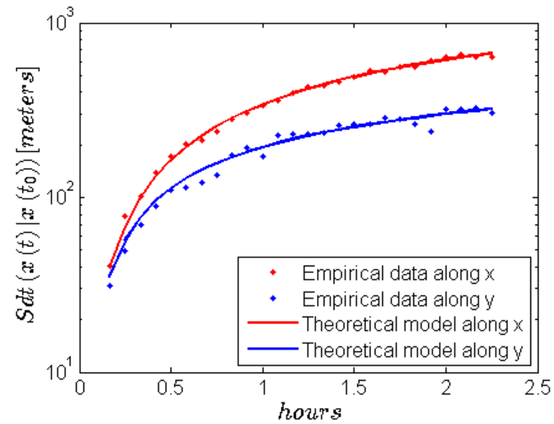


Fig. 10. Visualization of prediction error standard deviation along x and y for the tug route

*Example 3: Cargo/Tanker Route*

This section reports the results for a commercial route, made by 83% of cargoes and 17% of tankers.

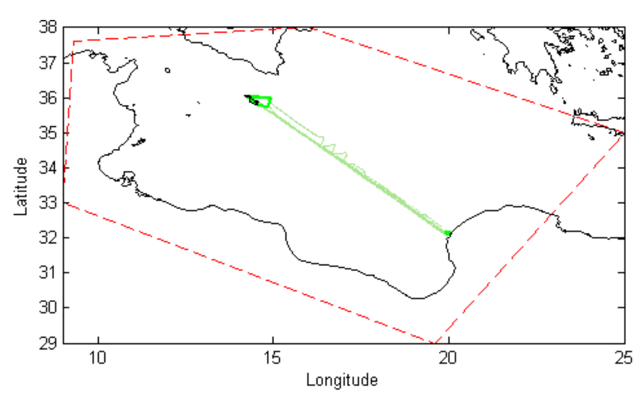


Fig. 11. Cargo/tanker route cluster: mean transit speed = 11.95 knots, route length = 641 km, median transit duration = 28.2 h.

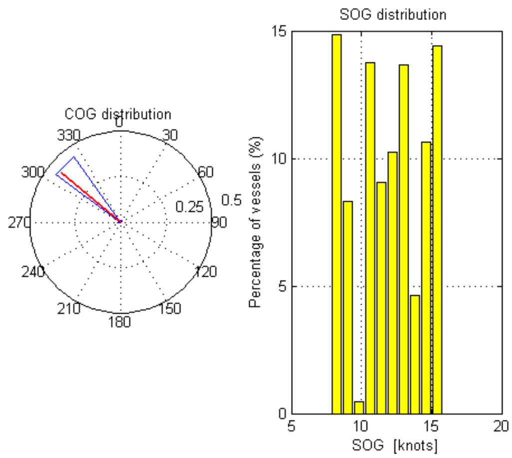


Fig. 12. Cargo/tanker route statistics: SOG distribution and COG distribution

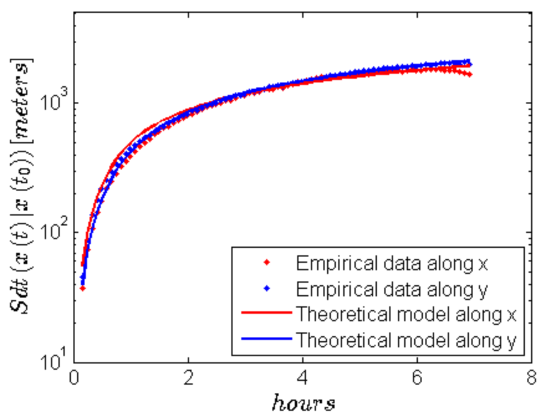


Fig. 13. Visualization of prediction error standard deviation along x and y for the cargo/tanker route

## V. CONCLUSION

A method has been presented for predicting motion patterns with a parametrized stochastic modelling method, enhanced by the historical traffic patterns derived in the area of interest. Specifically, the study has illustrated a potential exploitation of the traffic routes derived via TREAD.

The experimental results obtained using a real-world data set, in support of the second NEREIDS data campaign, have demonstrated the goodness of fit of the Ornstein-Uhlenbeck model for the uncertainty of vessel position predictions. This model was validated to estimate the position of vessels several hours ahead with an uncertainty of a few kilometres over a route on the order of hundreds of kilometres. However, the maximum time window for vessel prediction will depend on the route mean time duration, since longer routes provide a longer surveillance scenario.

A potential extension of the methodology will include multi-mode pdfs in proximity to waypoints and forks, where a probabilistic approach, able to model the switching to either one route or another, is needed.

As future work, the methodology will be explored for enhancing vessel destination prediction (*i.e.*, to provide a long-term forecast of vessel position to estimate the destination port, based on the current location or segment of a track). This will also then subsequently support destination validation from AIS messages. Another potential application of the method will be for new sensor deployment and imagery procurement plans in order to maximize Vessel of Interest (VOI) detection probability.

## ACKNOWLEDGEMENT

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